UNIVERSITÀ DI PISA
DOTTORATO DI RICERCA IN INGEGNERIA INDUSTRIALE

Doctoral Course

Spaceflight mission analysis in non-Keplerian gravitational systems

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Contents

The preliminary analysis of a space mission involves many specific engineering aspects, including the determination of a suitable trajectory and working orbit, the design of subsystems such as attitude determination and control, thermal control, and communication with ground stations. Therefore, every engineer who is currently working or is planning to work on space missions should be aware of how such scenarios are generated and what are their main characteristics.

The usual approach used for constructing space trajectories exploits patched conic arcs, each of which is a Keplerian solution of a two-body problem. However, the rise of low-thrust propulsion systems, such as electric engines and the innovative solar sails, has led to the development of more complex mathematical models, in which a continuous thrust affects the spacecraft for a long time. This has enabled the generation of a wide range of mission scenarios, otherwise unfeasible or highly costly. The Keplerian approach is also unable to identify some peculiarities of the gravitational fields, which can be analyzed only when two attractors are included in the model at the same time.

This course will start by introducing some basic concepts of the Keplerian approach, emphasizing its effectiveness and its limits. Then, an overview is given of non-Keplerian mission scenarios involving a single attractor, including low-thrust propelled trajectories and displaced non-Keplerian orbits. Subsequently, a second attractor will be included in the analysis, leading to the formulation of the circular restricted three-body problem (CR3BP). The latter model is analyzed in detail, focusing on the determination of equilibrium points and on orbits around those points (Lyapunov, Lissajous, Halo). The final part involves the description of a typical mission in the context of the CR3BP.

Schedule

The course is made up of six three-hour lectures, which will be held twice a week, and a final two-hour lecture. More details on the topics of each lecture can be found here below.

1. **General notes on Keplerian approach for mission design**: formulation of two-body problem and Kepler's solution. Some examples of preliminary mission design by means of Kepler's model (patched conics). Limits of the Keplerian approach.
2. **Beyond the Keplerian approach**: some notes on low-thrust trajectories. Displaced non-Keplerian orbits and artificial equilibrium points: dynamical equations, stability, and mission applications. (3 hours)
3. **The circular restricted three-body problem (CR3BP)**: definition, fundamental hypotheses, dynamical equations in dimensional and non-dimensional form, Lagrangian and Hamiltonian formulations. (3 hours)
4. **Equilibrium points of CR3BP**: determination of collinear and triangular equilibrium points, linear stability analysis. Energetic levels and Hill's regions. Realms of possible motion. (3 hours)
5. **Analysis of the linearized motion in the vicinity of a collinear point**. Two-dimensional case: determination of planar Lyapunov orbits and transit, non-transit and asymptotic trajectories. Extension to the three-dimensional case: general solution, Lissajous orbits and introduction to the special case of Halo orbits. (3 hours)
6. **Analysis of Halo orbits**. Determination of Halo orbits and associated constraints. Some generalities on **Floquet approach** for orbit determination: concepts of state transition matrix and monodromy matrix. Notes on Poincaré maps, invariant manifolds, and their possible exploitation. (3 hours)
7. **Concluding remarks**. Possibility of stabilizing an equilibrium point by means of a suitable thrust control. Possible extension to the elliptic case. General recap of course contents. Example of current and planned mission concepts that exploit non-Keplerian orbits and the CR3BP approach. (2 hours)

Timetable

All lectures will be held in room "DIA", Dipartimento di Ingegneria Civile e Industriale – sede di Ingegneria Aerospaziale, via G. Caruso 8, Pisa, and will be scheduled as follows.

Lecture number	Date	Time	Lecturer
1	28/01/2020 (Tuesday)	14:30–17:30	Lorenzo Niccolai
2	30/01/2020 (Thursday)	14:30–17:30	Lorenzo Niccolai
3	04/02/2020 (Tuesday)	14:30–17:30	Lorenzo Niccolai
4	06/02/2020 (Thursday)	14:30–17:30	Lorenzo Niccolai
5	11/02/2020 (Tuesday)	14:30–17:30	Lorenzo Niccolai
6	13/02/2020 (Thursday)	14:30–17:30	Lorenzo Niccolai
7	14/02/2020 (Friday)	14:30–16:30	Lorenzo Niccolai

About the lecturer

Lorenzo Niccolai currently is Research Assistant of Spaceflight Mechanics at the Department of Civil and Industrial Engineering, University of Pisa. His main research fields are mission analysis and astrodynamics. His main activities consist of the analysis of low-thrust trajectories and the determination of non-Keplerian mission scenarios, with a particular focus on innovative propellantless propulsion systems, such as solar sails and electric solar wind sails (e-mail: lorenzo.niccolai@ing.unipi.it).